

Optimal Power Allocation for Regenerative Relay over α - μ Fading Channel

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Abstract— In this paper outage and bit error rate (BER) performance of two-hop regenerative relay based wireless system over alpha mu (α - μ) fading channel is analysed. We have seen that relay operating with optimal power allocation always outperforms the uniform power allocation scenario. This has been established by simulation for outage and BER performance metrics over α - μ fading channel for various α and μ values.

Keywords— α - μ fading; regenerative relay system; power allocation; outage; bit error rate.

I. INTRODUCTION

In a mobile network, relaying schemes provide promising technologies to transmit at higher data rate to destinations farther away. The relay based communication is an active and vital area of research in wireless communication today. In [3] a low mobility cellular relaying system downlink have been considered where two mobile users are served by two neighbouring decode-and-forward relays. It is shown that the diversity of each user's received data signal can be improved by receiving data from multiple relays. A study of the role of cooperative relays has been done in [4] to provide and improve secure communication rates through DF strategies in a full-duplex multiple relay network with an eavesdropper. In [5] a maximum-likelihood (ML) decoder have been derived for the DF protocol with a setup of a single pair of source and destination with one relay. Investigation of the possible enhancement of cooperated system using decode and forward protocol have been carried out in [6] and found that for the distributed communication systems the relays can cooperate and form virtual MIMO systems. The

benefits of AF and DF cooperative relay for secure communication are investigated in [7] within Wyner's iretap channel. The capacity regions are investigated for two relay broadcast channels in [8] where relay links are incorporated into two-user broadcast channels to support user cooperation. In [9] the drawback of cooperative relaying has been addressed regarding its spectral inefficiency, and its channel over reservation compared to direct transmissions. Low-complexity cooperative diversity protocols have been developed and analyzed in [10] that combat fading induced by multipath propagation in wireless networks, the underlying techniques exploit space diversity available through cooperating terminals' relaying signals for one another. A novel and unified analysis is proposed in [11], which is based on the moment-generating function approach, to efficiently and accurately compute the higher order statistics of the channel capacity for AF multi-hop transmission over generalized fading channels. Constrained non-linear optimisation or non-linear programming attempts [13] to find a constrained minimum of a scalar function of scalar variables starting at an initial estimate.

The paper is organized as follows. In Section 2, the α - μ fading model and its Probability density function is briefly discussed. In Section 3, the regenerative relay based wireless system has been analysed. In Section 4, uniform and optimum power allocation in two-hop relay system is described. Monte-Carlo simulation results for outage probability and BER performance of regenerative two-hop relay based wireless system over α - μ fading channel with varying α and μ parameters are presented in Section 5. The paper is concluded by Section 6.

II. THE α - μ FADING MODEL

The multipath fading in wireless communication is modeled by several distributions such as Rayleigh, Rician, Weibull, and Nakagami. In the recent past α - μ fading model [1] has been proposed to describe the mobile radio signal considering two important phenomenon of radio propagation non-linearity and clustering. The α - μ represents a generalized fading distribution for small-scale variation of

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the fading signal in a non line-of-sight fading condition. As given in its name, alpha-mu distribution is written in terms of two physical parameters, namely α and μ . The power parameter ($\alpha > 0$) is related to the non-linearity of the environment i.e. propagation medium, whereas the parameter ($\mu > 0$) is associated to the number of multipath clusters.

Table-1: Algorithm for generation of α - μ distributed random variable

1.	Procedure α - μ random variable generation
2.	$\alpha \leftarrow$ Channel Parameter
3.	$\mu \leftarrow$ Channel Parameter
4.	$x \leftarrow$ Number of random variables
5.	$\Omega \leftarrow$ Mean
6.	$\sigma^2 \leftarrow$ Variance
7.	$H \leftarrow$ zero matrix of order $1 \times x$
8.	for $i \leftarrow 1$ to μ do
9.	$H = H +$ matrix of order $1 \times x$ having complex Gaussian random variable i.e. $X(\Omega, \sigma^2) + j X(\Omega, \sigma^2)$
10.	end for
11.	Fading envelope $\leftarrow H^\alpha$
12.	end procedure

In [1, 2] the α - μ fading distribution and its probability density function has been described. In the α - μ distribution, it is considered that a signal is composed of clusters of multipath waves. In any one of the cluster, the phases of the scattered waves are random and have similar delay times. Further, the delay-time spreads of different clusters is generally relatively large. As a result, the obtained envelope is a non-linear function of the modulus of the sum of the multipath components. The α - μ probability density function (PDF), $f_R(r)$ of envelope R is given as

$$f_R(r) = \frac{\alpha \mu^\mu \Gamma^{\alpha\mu-1}}{\hat{r}^{\alpha\mu} \Gamma(\mu)} \exp\left[-\mu \frac{r^\alpha}{\hat{r}^\alpha}\right] \quad (1)$$

where $\alpha > 0$ is the power parameter, and α -root mean value of R^α is given as

$$\hat{r} = \alpha \sqrt[\alpha]{E(R^\alpha)} = \alpha \sqrt[2]{2\mu\sigma^2}$$

where $\mu \geq 0$, is inverse of variance of α - μ envelope R^α , and

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \quad \text{is the Gamma function.}$$

Algorithm given in Table.1 is used for generation of α - μ distributed random variable in the simulation work reported in this paper. Analytical and simulated results for PDF of fading envelope of α - μ fading channel defined by eq. (1), are shown in Fig.1. It is verified that both the analytical and simulated results are matching.

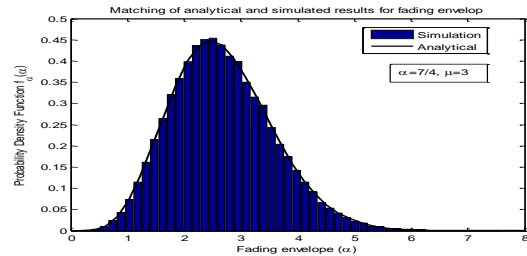


Fig.1. Matching of analytical and simulated results for α - μ fading envelope

PDF of SNR (γ) of α - μ fading is given in [12] as

$$f_\gamma(\gamma) = \frac{\alpha \mu^\mu \gamma^{\frac{\alpha\mu}{2}-1}}{2 \Gamma(\mu) \bar{\gamma}^{\alpha\mu/2}} e^{-\mu \left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} \quad (2)$$

III. REGENERATIVE RELAY SYSTEM

In a two-hop wireless communication system with relay shown in Fig.2, the signal is transmitted as broadcast through relay in between source and destination. Each terminal is equipped with single transmit/receive antenna.

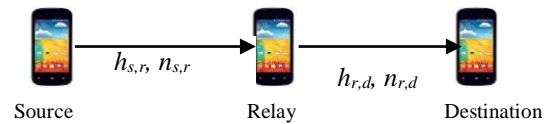


Fig.2. Two-hop wireless communication system with relay

The regenerative system is basically decode and forward (DF) digital scheme, where relay receives the signal, decodes it and after encoding retransmit it to the destination. Noise does not propagate, because the noise will not be amplified and it is excluded by the decoding process. However, when a decoding error occurred at the relay node due to the deep fading in channel between the source and the relay, this can be considered as the major problem with decode and forward method. The problem will be worsen if detection at the relay node is also unsuccessful and will result in bad performance. Due to the broadcast nature of the wireless medium, the relay and the destination nodes will receive noisy copy of the signals. In DF method processing time is higher causing delay, hence DF is not suitable for delay sensitive signals.

Assuming that source transmitting signal x , which is received by relay as $y_{s,r}$ and is given by (3).

where

$h_{s,r}$ & $h_{r,d}$ = fading amplitude with PDF defined by (1) by

replacing R with $h_{s,r}$ & $h_{r,d}$ for source-to-relay &

relay-to-destination $\alpha - \mu$ wireless channels respectively.

$n_{s,r}$ & $n_{r,d}$ = Additive White Gaussian Noise (AWGN) of the

$\alpha - \mu$ wireless channels; source-to-relay & relay-to-destination respectively with variance σ^2 .

The signal received at relay is decoded and encoded.

Let the encoded signal be \hat{x} .

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$$\gamma_{s,r} = \frac{h_{s,r}^2 P_s}{\sigma^2} = \delta_{s,r} P_s \tag{6}$$

$$\gamma_{r,d} = \frac{h_{r,d}^2 P_r}{\sigma^2} = \delta_{r,d} P_r \tag{7}$$

where $\delta_{s,r} = \frac{h_{s,r}^2}{\sigma^2}$, and $\delta_{r,d} = \frac{h_{r,d}^2}{\sigma^2}$,

P_s and P_r are the transmit power at source and relay respectively. The, SNR at destination for DF relay link will be

$$\gamma_{DF} = \min(\gamma_{s,r}, \gamma_{r,d}) \tag{8}$$

The CDF of SNR for source-relay-destination DF link is

$$F_{\gamma_{DF}}(\gamma) = 1 - \left[\{1 - F_{\gamma_{sr}}(\gamma)\} \{1 - F_{\gamma_{rd}}(\gamma)\} \right] \tag{9}$$

Assuming both links source-relay and relay-destination identical, for simplicity in analysis, hence CDF for both link equal to $F_{\gamma}(\gamma)$, therefore we get

$$F_{\gamma_{DF}}(\gamma) = 1 - \left[\{1 - F_{\gamma}(\gamma)\}^2 \right] \tag{10}$$

With help of PDF given in (2), the CDF is obtained

$$F_{\gamma}(\gamma_{th}) = \int_0^{\gamma_{th}} f_{\gamma}(\gamma) d\gamma = \int_0^{\gamma_{th}} \frac{\alpha \mu^{\alpha} \gamma^{\alpha\mu - 1}}{2 \Gamma(\mu) \bar{\gamma}^{\alpha\mu/2}} e^{-\mu \left(\frac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}} d\gamma \tag{11}$$

On solving (11), we get CDF [12] for identical links as

$$F_{\gamma}(\gamma_{th}) = \frac{\gamma \left(\mu, \mu \left(\frac{\gamma_{th}}{\gamma} \right)^{\alpha/2} \right)}{\Gamma(\mu)} \tag{12}$$

Now, CDF of SNR for source-relay-destination DF link, from (10) will be

$$F_{\gamma_{DF}}(\gamma_{th}) = 1 - \left[\left[1 - \frac{\gamma \left(\mu, \mu \left(\frac{\gamma_{th}}{\gamma} \right)^{\alpha/2} \right)}{\Gamma(\mu)} \right]^2 \right] \tag{13}$$

This will be obtained by maximum likelihood detector, when signal x is estimated by relay.

$$\hat{x} = \arg \min_x |y_{s,r} - h_{s,r}x|^2 \tag{4}$$

Hence, the signal received in DF, at destination will be

$$y_{r,d} = h_{r,d} \hat{x} + n_{r,d} \tag{5}$$

The SNR (γ) for each link is defined as below

The CDF obtained in (13) is also the outage probability for source-relay-destination DF link. Therefore

$$P_{out|DF} = 1 - \left[\left[1 - \frac{\gamma \left(\mu, \mu \left(\frac{\gamma_{th}}{\gamma} \right)^{\alpha/2} \right)}{\Gamma(\mu)} \right]^2 \right] \tag{14}$$

CDF of (13), when differentiated will result in PDF of source-relay-destination DF link. Therefore, the BER will be given as

$$P_e|_{DF} = \int_0^{\infty} Q(a\sqrt{\gamma}) f_{\gamma_{DF}}(\gamma) d\gamma \tag{15}$$

where a is a constant and depends on modulation and detection combination.

IV. POWER ALLOCATION

Here we formulate the problem, which will be investigated through simulation. Scenario of a network is considered, in which individual node has upper bound of energy (i.e. limited power wireless terminal).

$$0 \leq P_s \text{ \& \ } 0 \leq P_r \tag{16}$$

These terminals works in network of cellular type or works in unlicensed band. Total power radiated by network in this band should not exceed beyond specified level to avoid interfering other networks work in this band, else the total network is battery powered whose total power is known. Such types of network have upper bound on total power transmission. So transmitted power by source is given by

$$P_s = \lambda P_T; \quad 0 < \lambda < 1 \tag{17}$$

Here P_T is total available power in this network. Transmitted power by supporting relay is given by

$$P_r = (1 - \lambda) P_T \quad (18)$$

This is the simplest schemes and supporting relay placed at middle gives optimum capacity gain. **Optimum Power Allocation:**

Optimum power allocation is a centralized power allocation technique in which source should have full CSI between all nodes prior to transmission. When each node ¹Dharmraj, ²Himanshu Katiyar

estimate their source-to-relay CSI, due to broadcast nature of the wireless medium. Similarly when relay (r) transmits the training bits, the CSI of source-to-relay and relay-to-destination can be estimated at source and destination (d) respectively. (We assume that forward and backward channels between relay and destination are the same due to reciprocity. These transmission occur on the same frequency band and same coherence interval). However, CSI of relay-to-destination can be available at the source only through channel feedback.

In a slow fading environment, frequent training is not necessary. Hence, in this case we can neglect training period as compared to actual data transmission period. On the basis of CSI i.e. (γ_{sr}, γ_{rd}), source distributes the available power (P_T) between s and r . So, our primary objective is effectively utilizing the available power to boost γ_T at destination i.e.

$$\max_{P_s, P_r} \gamma_T = \min(\delta_{sr}, P_s, \delta_{rd}, P_r) \quad (20)$$

$$\text{subject to } \begin{cases} P_s + P_r = P_T, \\ 0 \leq P_s, \\ 0 \leq P_r, \end{cases} \quad (21)$$

V. SIMULATION RESULTS AND DISCUSSIONS

Outage and BER performance for two-hop regenerative relay based system over α - μ fading channel is obtained by Monte-Carlo simulation. The communication system shown

A. Uniform Power Allocation:

In traditional uniform power allocation, power is equally distributed between source (s) and relay (r), without taking channel state information (CSI) into account.

$$\text{operate } P_s = \frac{1}{2} P_T, \text{ and } P_r = \frac{1}{2} P_T \quad (19)$$

channels are estimated by sending training sequence before the actual message transmission. When the source (s) transmits the training bits, all relay node can simultaneously

in fig.2 has been considered in this simulation. Outage probability and BER performance simulated results for power optimized and un-optimized for different α and μ values are shown in fig. 3 to fig.10. In these simulation 2000 bits have been considered for a particular α and μ combination. The total available power P_T is equal to $2 \times$ average SNR.

In fig.3 the outage performance for $\alpha=1, \mu=1$ is shown. It is seen that at 15 dB SNR, the outage for uniform power allocation case is higher as $\approx 10^{-0.6}$, and that for optimized case is $\approx 10^{-0.7}$. In fig.5, α is increased to 2 and μ parameter is same as in fig.4, outage for uniform power case is 8.0×10^{-2} , and for optimized case is 5.0×10^{-2} thus we find an improvement in outage with increase in α value.

Further improvement in outage is noticed in fig.7 with increase in α equal to 3 and $\mu=1$, here it is noticed, at 15 dB SNR, the outage for uniform power case is 2.3×10^{-2} , and for optimized case is 1.3×10^{-2} . In fig.9 with $\alpha=1, \mu=2$, the outage for uniform power case is observed as 2.2×10^{-2} , and for optimized case is 1.7×10^{-2} .

In fig.4 the BER performance for α & μ equal to 1 is shown. It is seen that at 15 dB SNR, the BER for uniform power case is higher as $\approx 10^{-1}$, and that for optimized case is 8.0×10^{-2} . In fig.6, the BER performance for $\alpha=2$ and $\mu=1$, it is seen that at 15 dB SNR, the BER for uniform power case is 2.5×10^{-2} , and that for optimized case is 1.6×10^{-2} . Fig.8 shows the BER for $\alpha=3$ and $\mu=1$, it is found that at 15 dB SNR, the BER for uniform power case is 1.5×10^{-2} , and that for optimized case is 4.0×10^{-3} . In fig. 10, for $\alpha=1, \mu=2$, BER for uniform power case is noted as 1.2×10^{-2} , and that for optimized case is 4.0×10^{-3} .

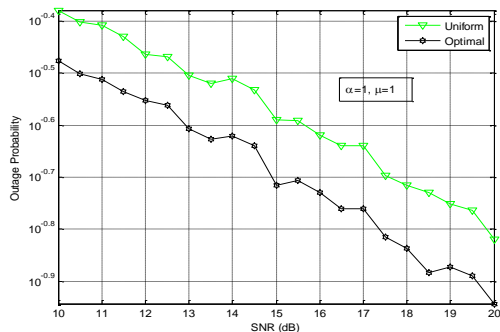


Fig. 3. Outage of power allocated two-hop regenerative relay over α - μ fading for $\alpha=1, \mu=1$

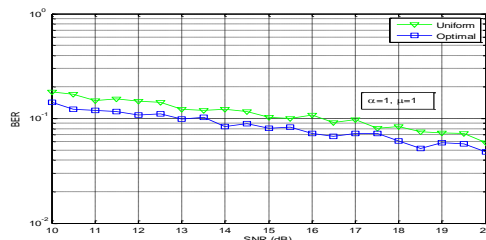


Fig. 4. BER of power allocated two-hop regenerative relay over α - μ fading for $\alpha=1, \mu=1$

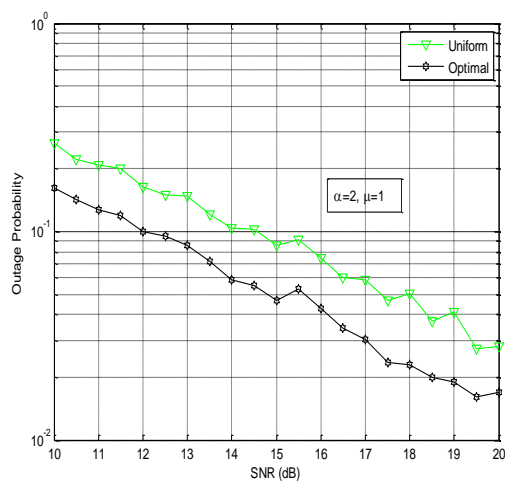


Fig. 5. Outage of power allocated two-hop regenerative relay over α - μ fading for $\alpha=2, \mu=1$

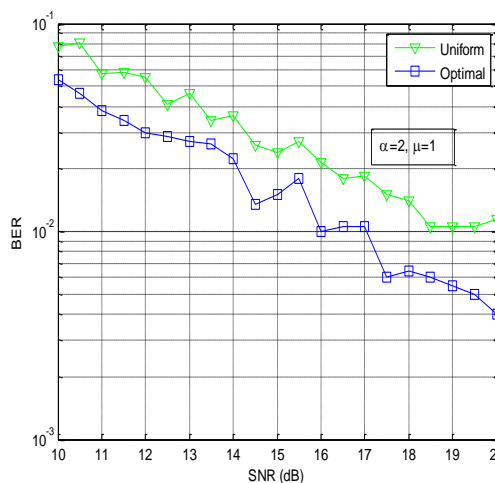


Fig. 6. BER of power allocated two-hop regenerative relay over α - μ fading for $\alpha=2, \mu=1$

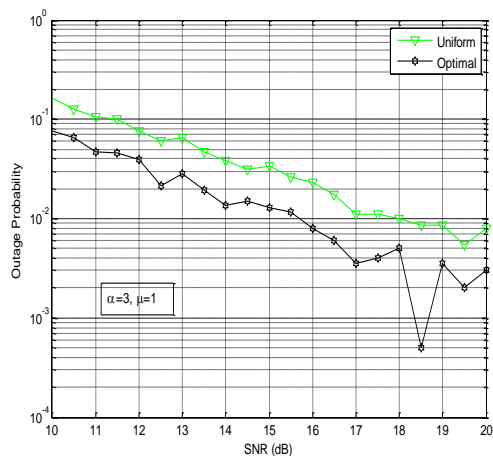


Fig. 7. Outage of power allocated two-hop regenerative relay over α - μ fading for $\alpha=3, \mu=1$

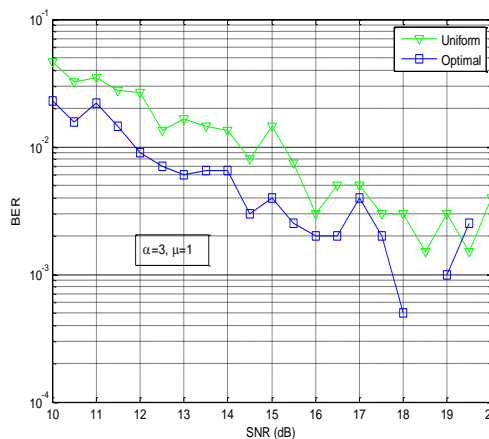


Fig. 8. BER of power allocated two-hop regenerative relay over α - μ fading for $\alpha=3, \mu=1$

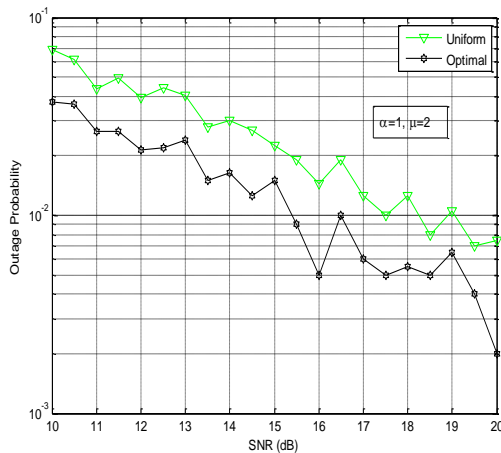


Fig. 9. Outage of power allocated two-hop regenerative relay over α - μ fading for $\alpha=1, \mu=2$

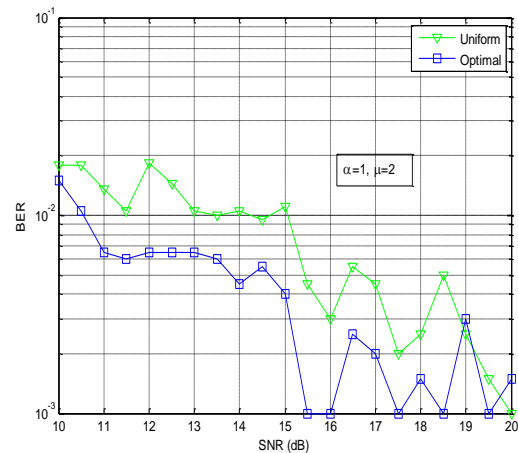


Fig. 10. BER of power allocated two-hop regenerative relay over α - μ fading for $\alpha=1, \mu=2$

VI. CONCLUSION

In this paper outage probability and BER performance of two-hop relay based wireless system, have been analysed for uniform power allocation and optimal power allocation condition over α - μ fading channel. The performance of regenerative relay system is compared with simulation results for uniform and optimal power allocation scenario. It has been analyzed that with increase in α and μ parameters, BER and outage performance improves. The power optimized relay based system outperforms the uniform power system. This analysis will be helpful for further investigation on optimization of power of cooperative wireless systems with multi-hop over α - μ fading channel.

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